Question number	Scheme	Marks
1.	$4\left(1-\text{sech}^2 x\right)-2\,\text{sech}^2 x=3$	M1
	$6\,\text{sech}^2 x=1$	A1
	$\cosh x=\sqrt{6}$	M1 A1
	Using $\text{arcosh}\, x=\ln\left(x+\sqrt{\left(x^2-1\right)}\right)$	M1
	$x=\pm\ln\left(\sqrt{6}+\sqrt{5}\right)$	A1 (6)
		(6 marks)
2.	$\frac{\mathrm{d}y}{\mathrm{d}x}=\sinh\left(\frac{1}{2}x\right)$	B1
	$\int y\sqrt{\left(1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)}\mathrm{d}x = \int 2\cosh\left(\frac{1}{2}x\right)\sqrt{\left(1+\sinh^2\left(\frac{1}{2}x\right)\right)}\mathrm{d}x$	M1 A1
	$=\int 2\cosh\left(\frac{1}{2}x\right)\mathrm{d}x$	
	$=\int\left(1+\cosh x\right)\mathrm{d}x$	
	$=x+\sinh x$	M1 A1
	$2\pi\left[x+\frac{\mathrm{e}^x-\mathrm{e}^{-x}}{2}\right]_{-\ln 2}^{\ln 2} = \pi\left[\left(2\ln 2+2-\frac{1}{2}\right)-\left(-2\ln 2+\frac{1}{2}-2\right)\right]$	M1
	$=\pi\left(4\ln 2+3\right)$	A1 (7)
		(7 marks)

Question number	Scheme	Marks
3.	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -\frac{3\cosh\theta}{\sinh^2\theta}$	B1
	$\int \frac{1}{x\sqrt{(x^2+9)}}\,\mathrm{d}x = \int \frac{1}{\frac{3}{\sinh\theta}\sqrt{\left(\frac{9}{\sinh^2\theta}+9\right)}} \times \frac{-3\cosh\theta}{\sinh^2\theta}\,\mathrm{d}\theta$	M1 A1
	$= -\frac{1}{3}\int 1\,\mathrm{d}\theta = -\frac{1}{3}\theta$	A1
	$x = 3\sqrt{3} \Rightarrow \sinh\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \ln\left(\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}\right) = \ln\sqrt{3}$	
	$x = 4 \Rightarrow \sinh\theta = \frac{3}{4} \Rightarrow \theta = \ln\left(\frac{3}{4} + \frac{5}{4}\right) = \ln 2$	M1 A1
	$\left[-\frac{1}{3}\theta\right]_{\ln 2}^{\ln\sqrt{3}} = \frac{1}{3}\left(\ln 2 - \ln\sqrt{3}\right) = \frac{1}{3}\left(\frac{1}{2}\ln 4 - \frac{1}{2}\ln 3\right) = \frac{1}{6}\ln\frac{4}{3}$	M1 A1 (8)
		(8 marks)
4. (a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1+x} \times \frac{1}{2}x^{-\frac{1}{2}} \left(= \frac{1}{2x^{\frac{1}{2}}(1+x)}\right)$	M1 A1
	$x = \frac{1}{4} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{5}$	A1 (3)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}(1+x)^{-1}x^{-\frac{1}{2}}$	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\frac{1}{2}(1+x)^{-2} \times x^{-\frac{1}{2}} - \frac{1}{4}(1+x)^{-1} \times x^{-\frac{3}{2}}$	M1 A1
	$= -\frac{1+3x}{4x^{\frac{3}{2}}(1+x)^2}$	
	$2x(1+x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + (1+3x)\frac{\mathrm{d}y}{\mathrm{d}x}$	
	$= 2x(1+x)\left(-\frac{1+3x}{4x^{\frac{3}{2}}(1+x)^2}\right) + (1+3x)\left(\frac{1}{2x^{\frac{1}{2}}(1+x)}\right)$	M1 A1, A1
	$= 0$ ✱	A1 cso (6)
		(9 marks)

Question number	Scheme	Marks
5. (a)	$I_n = \int_0^{\frac{\pi}{2}} \sin x . \sin^{n-1} x \, dx$	
	$= \left[-\cos x \sin^{n-1} x \right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x . (n-1) \sin^{n-2} x \cos x \, dx$	M1 A1
	$= 0 + \ ...$	A1
	$= (n-1) \int_0^{\frac{\pi}{2}} \sin^{n-2} x \left(1 - \sin^2 x \right) dx$	
	$= (n-1) I_{n-2} - (n-1) I_n$	M1
	Leading to $I_n = \frac{n-1}{n} I_{n-2}$ (✱)	A1 (5)
(b)	$\int_0^{\frac{\pi}{2}} x \left(\sin^5 x \cos x \right) dx = \left[\frac{x \sin^6 x}{6} \right]_0^{\frac{\pi}{2}} - \frac{1}{6} \int_0^{\frac{\pi}{2}} \sin^6 x \, dx$	M1 A1
	$I_6 = \frac{5}{6} \times \frac{3}{4} \times \frac{1}{2} I_0 = \frac{5}{6} \times \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2} \left(= \frac{5\pi}{32} \right)$	M1 A1
	Hence $\int_0^{\frac{\pi}{2}} x \left(\sin^5 x \cos x \right) dx = \frac{\pi}{12} - \frac{1}{6} \times \frac{5\pi}{32} = \frac{11\pi}{192}$	A1 (5)
		(10 marks)

Question number	Scheme	Marks
6. (a)	$\overrightarrow{PQ}=\begin{pmatrix}-3\\4\\-4\end{pmatrix}$, $\overrightarrow{QR}=\begin{pmatrix}2\\2\\-2\end{pmatrix}$	B1
	$\overrightarrow{PQ}\times\overrightarrow{QR}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-3&4&-4\\2&2&-2\end{vmatrix}=\begin{pmatrix}0\\-14\\-14\end{pmatrix}$	M1 A2, 1, 0 (4)
(b)	$\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}0\\1\\1\end{pmatrix}=\begin{pmatrix}1\\-3\\1\end{pmatrix}.\begin{pmatrix}0\\1\\1\end{pmatrix}=-2 \quad\Rightarrow\quad y+z=-2$ or equivalent	M1 A1 (2)
(c)	$y+z=-2$	
	$x+y-z=6$	
	Let $z=\lambda \quad\Rightarrow y=-\lambda-2,\ x=2\lambda+8$	M1 A1 A1
	$\mathbf{r}=\begin{pmatrix}8\\-2\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\1\end{pmatrix}$	M1
	$\left(\mathbf{r}-\begin{pmatrix}8\\-2\\0\end{pmatrix}\right)\times\begin{pmatrix}2\\-1\\1\end{pmatrix}=\mathbf{0}$	A1 (5)
		(11 marks)

Question number	Scheme	Marks
7. (a)	$\begin{pmatrix} 2 & k & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}\begin{pmatrix} 9 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 3k+18 \\ 12 \\ -8 \end{pmatrix} = \lambda\begin{pmatrix} 9 \\ 3 \\ -2 \end{pmatrix}$	M1
	$\lambda = 4 \quad \Rightarrow \quad 3k+18=36 \quad \Rightarrow \quad k=6$	M1 A1 (3)
(b)	$\lambda = 4$ is an eigenvalue	B1
	$\begin{vmatrix} 2-\lambda & 6 & 0 \\ 1 & 1-\lambda & 0 \\ 0 & -2 & 1-\lambda \end{vmatrix} = (1-\lambda)\left((2-\lambda)(1-\lambda)-6\right)$	M1
	$(\lambda-1)(\lambda^2-3\lambda-4) = (\lambda-1)(\lambda-4)(\lambda+1)$	M1
	$\lambda = (4,)\ 1,\ -1$ 1 and -1	A1 (4)
(c)	$\begin{pmatrix} 2 & 6 & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}\begin{pmatrix} t-2 \\ t \\ 2t \end{pmatrix} = \begin{pmatrix} 8t-4 \\ 2t-2 \\ 0 \end{pmatrix}$	M1 A2,1,0
	$x = 8t-4,\ y = 2t-2,\ z = 0$	
	$x - 4y - 4 = 0$	M1 A1 (5)
		(12 marks)

Question number	Scheme	Marks
8. (a)	$\frac{dx}{du}=5\sec u\tan u,\ \frac{dy}{du}=3\sec^2 u$	
	$\frac{dy}{dx}=\frac{3\sec^2 u}{5\sec u\tan u}\quad\left(=\frac{3}{5\sin u}\right)$	B1
	$y-3\tan u=\frac{3}{5\sin u}(x-5\sec u)$	M1
	Leading to $3x=5y\sin u+15(\sec u-\tan u\sin u)$	
	$3x=5y\sin u+15\left(\frac{1-\sin^2 u}{\cos u}\right)$	
	$3x=5y\sin u+15\cos u$ (✱)	M1
		A1 cso (4)
(b)	Equations of asymptotes $y=\pm\frac{3}{5}x$ both	B1
	Eliminating y or x between $3x=5y\sin u+15\cos u$ and $y=\frac{3}{5}x$	
	$3x=3x\sin u+15\cos u$	
	$x=\frac{5\cos u}{1-\sin u},\quad y=\frac{3\cos u}{1-\sin u}$	M1 A1
	Similarly between $3x=5y\sin u+15\cos u$ and $y=-\frac{3}{5}x$	
	$x=\frac{5\cos u}{1+\sin u},\quad y=-\frac{3\cos u}{1+\sin u}$	M1 A1
	Let (x_M, y_M) be the coordinates of the mid-point of RS.	
	$x_M=\frac{1}{2}\left(\frac{5\cos u}{1-\sin u}+\frac{5\cos u}{1+\sin u}\right)=\frac{5\cos u}{2}\left(\frac{2}{1-\sin^2 u}\right)=5\sec u$	M1
	$y_M=\frac{1}{2}\left(\frac{3\cos u}{1-\sin u}-\frac{3\cos u}{1+\sin u}\right)=\frac{3\cos u}{2}\left(\frac{2\sin u}{1-\sin^2 u}\right)=3\tan u$	A1
	The coordinates (x_M, y_M) are the same as P.	
	P is the mid-point of RS. (✱)	A1 cso (8)
		(12 marks)